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CDM

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The component method could be considered as a generalization of the model described by the growth balance technique, using the age as the basic variable.

$$N^t = N^0 + B - D + I - E$$

Where:

N^t = population in the time t

N^0 = population in t = 0 or also known as the base population

B = number of births between moments t=0 and t=t

D = number of deaths during the period t=0 and t=t

I = number of migrants between t=0 and t=t

E = people that left between t=0 and t=t

It is important to make clear that population projections determined by using the component method, assumed that the changes in size, composition and geographical distribution of the population depend on the mortality, fertility and migration trends.

Mortality Projection

The main objective in the determination of the future mortality is to derive the future survival rates that will be later use to elaborate the population projection. Based on the relative stability of the mortality trends, the future projections of population are less risky than the fertility trends. A mortality projection requires, on one side, to project the level (life expectancy at birth) and on the other side, to project the structure (mortality rates by age).

Life Expectancy Projection

The life expectancy projection is estimated with the use of the following logistic function

$$e_0(t) = \frac{K}{1+e^{a+bt}}$$

Where:

$e_0(t)$ life expectancy at birth in moment t t
K maximum limit of life expectancy
a, b parameters
t time

Mortality projection by age

The mortality by age depends on the changes in the population level of the mortality.

$$Y(x) = \alpha + \beta * Y^s(x)$$

The Brass two parameters model is a mathematical function that uses a mortality function by age that it is assumed standard with the mortality conditions of other periods of time. The main equation of the model can be written as:

$$Y(x) = \log ito(1 - \ell_x) = \tfrac{1}{2} \ell n\left(\frac{1 - \ell_x}{\ell_x}\right)$$

Where y is given by:

$$Y^s(x) = \log ito(1 - \ell_x^s)$$

The parameters α, associated to the level of the standard, and β, associated with the slope of the standard, make this function linear for tow mortality conditions in two different moments. When the limit mortality is selected, the mortality of moment in time having a standard population, their respective survivorship probabilities can be calculated with the application of this method for each year.

Fertility Projection

The procedure of the fertility rates by ages is based on the assumption of the behavior of future fertility rates trends. Representative fertility rates are required for a period of 5 years in order to provide an average number of births, that multiplied by 5 will give the total births of the period.

- Fertility rates for different assumed ages periods t, t+5

$${}_5 f_x^{t,t+5}$$

- Female population that are (x, x+4) years old at the instant t

$${}_5 N_x^{F,t}$$

- Female population that are (x, x+4) years old (survivals +migration) at t+5

$${}_5 N_x^{F,t+5}$$

- Total births corresponding to the period t, t+5

$B^{t,t+5}$
If it is assumed that women younger than 15 and older the 49 cannot conceive, it is found that:

$$B^{t,t+5} = 5\sum_{15}^{45}\left[{}_5 f_x^{t,t+5} * \frac{{}_5N_x^{F,t} + {}_5N_x^{F,t+5}}{2} \right] \forall x = 5,10.....30$$

To distinguish between the male and females births (100 women for every 205 male births 0.4878 IM 1.05) assuming the masculinity index at birth.

Fertility Projection Mechanics

It is done in two stages; first, the general fertility is projected in terms of the TCF or in terms of the TBR. Later, the relative population projetcted fertility is done in tow stages: first of all in terms of the TGF and the TBR or the relative distribution by groups of ages. Finally, this distribution is applied in terms of the TGF or in terms of the TBR, later the relative population projections is estimated by ages, finally it is applied to the distribution to the TGF and that’s how the projected population rates for different groups of age are obtained.

One of the general criteria to project fertility is based on the demographic transition theory, that assumes this variable will tend to decrease, in the long term, to a point in which, it will have to reemployed. The decrease of this variable can be seen as a logistic type behavior.

$$TGF(t) = K_1 + \frac{K_2}{1 + e^{a+bt}}$$

Where:

TGF(t)	global fertility rate at the instant t
$K_1 + K_2$	superior asymptote
K_1	inferior asymptote
a, b	parameters
t	time

The following values are usually defined:

$K_1 + K_2$ highest TGF (observed or supposed), for the country in a past time (long past time)

K_1 final value of the TGF in the transition process

TGF(0) TGF at the start time of the projection

TGF(T) TGF corresponds to an instant T in the future

with the above values, a and b can be calculated solving the following equation system:

$$a = \ln\left[\frac{K_1 + K_2 - TGF(0)}{TGF(0)}\right]$$

$$b = \frac{1}{T}\left[\ln\left[\frac{K_1 + K_2 - TGF(T)}{TGF(T) - K_1}\right] - a\right]$$

Fertility structure projection by age

TGF can be interpreted as the average number of sons / daughters that a woman can conceive at the end of its fertility reproductive period in a hypothetic cohort of woman that have been influenced at a certain time by the fertility by ages. In the hypothesis that not being in that fertility group, at each age, for each women, the considered cohort, and also assuming the mortality of women is zero at the end of their reproductive period.

The relative distribution of fertility rates by ages indicates the way in which women have been having children in their lifetime. For instance, it is possible to find for certain level of fertility (given by the TGF) different fertility curves by age. The common procedure that is followed is to project the general level of the TGF, keeping

$$F(x) = (TGF)A^{B^x}$$

in mind to obtain a coherent result of it; it is projected not depending on the fertility structure by ages. Using the Gompertz function, the accumulated fertility can be represented as:

where:

x age

F(x) accumulated fertility at age x

TGF value of F(x) for the superior female limit of age to conceive

A y B positive parameters less than 1

The relative accumulated distribution takes the following relation

$$\frac{F(x)}{TGF} = A^{B^x}$$

Applying the natural logarithm twice, we obtain:

$$\ln\left[-\ln\frac{F(x)}{TGF}\right] = x\ln B + \ln(-\ln A)$$

With these equations a linear graph is constructed.

Once the vector has been obtained, the estimation of the parameters a and b, the fertility projection of the structure of the fertility is estimated.